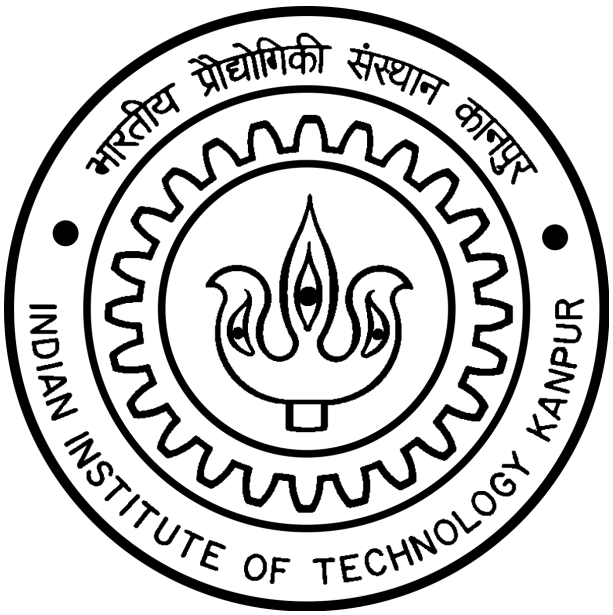

MATHEMATICS AND SCIENTIFIC COMPUTING

PROJECT REPORT:

NON LINEAR REGRESSION

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Contents

1 Introduction

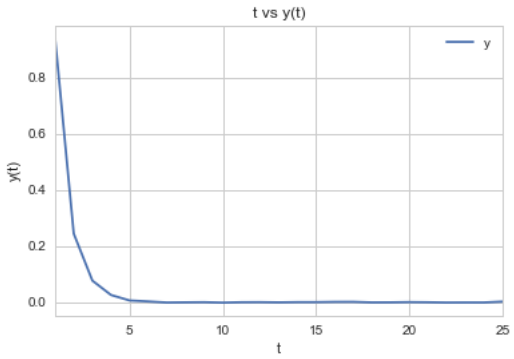
In this Project, we have been given some data which has non-liner relationship, using this data we have to estimate the Non-linear model parameters. The Non-linear model is the following:

$$y(t) = \alpha_1 e^{\beta_1 t} + \alpha_2 e^{\beta_2 t} + \epsilon(t) \tag{1}$$

Here $\epsilon(t)$ is a sequence of i.i.d. random variable with mean zero and finite variance.

2 Solution of the given problem

2.1 Plot the data

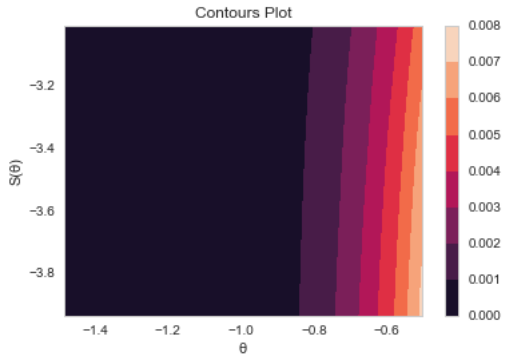


2.2 Can you find real solution using Prony's equations

Using Prony's algorithm I got the following estimated value of the Non-linear model parameters:

- α_1 : 2.934109
- α_2 : 1.976207
- β_1 : -1.246141
- β_2 : -3.032437

2.3 Plot the contours



2.4 Estimate the unknown parameters using Newton Raphson or Gauss Newton method

Using Gauss Newton's algorithm I got the following estimated value of the Non-linear model parameters:

- α_1 : 2.343090
- α_2 : 10.471415
- β_1 : -1.137324
- β_2 : -4.020549

2.5 Estimate the unknown parameters using Osborne's method

Using Osborne's algorithm I got the following estimated value of the Non-linear model parameters:

- α_1 : 2.965406
- α_2 : 5.315211
- β_1 : -1.245917
- β_2 : -4.122295

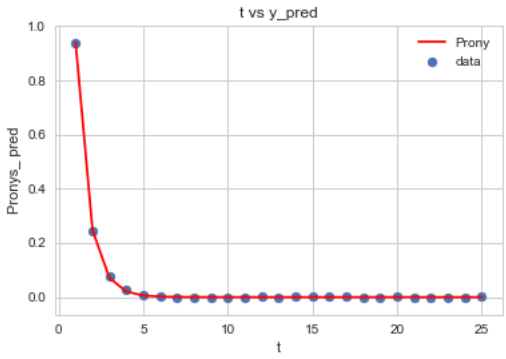


Figure 1: Prony

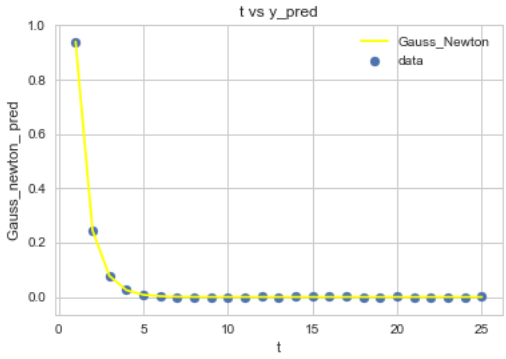


Figure 2: Gauss Newton

2.6 Plot the predicted curve

- Fig1: Y predicted by Prony
- Fig2: Y predicted by Gauss Newton
- Fig3: Y predicted by Osborne
- Fig4: Y predicted by All method in one graph

2.7 Plot the errors

- Fig5: Error in Prony
- Fig6: Error in Gauss Newton
- Fig7: Error in Osborne

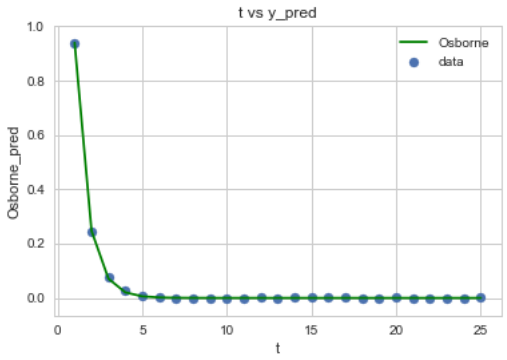


Figure 3: Osborne

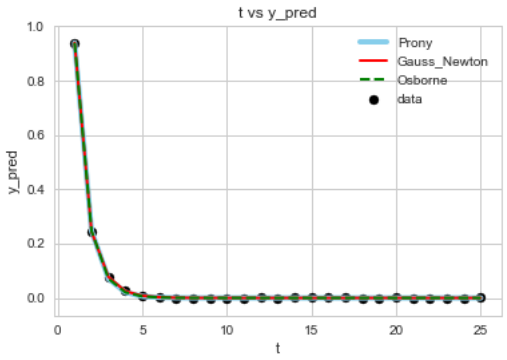


Figure 4: y Pred

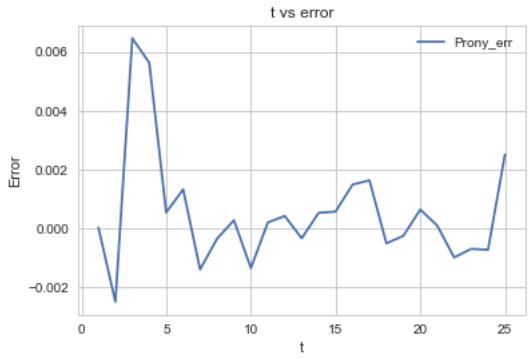


Figure 5: Prony Error

Figure 6: Gauss Newton Error

- Fig8: Error in All method in one graph

2.8 Find the confidence intervals of the unknown parameters. the number of components using cross validation approach

2.8.1 CI of Parameters estimated by Prony's method

- CI of $\alpha_1 = (0.870210, 4.998008)$
- CI of $\alpha_2 = (-0.087691, 4.040105)$
- CI of $\beta_1 = (-3.310039, 0.817757)$
- CI of $\beta_2 = (-5.096335, -0.968537)$

Figure 7: Osborne Error

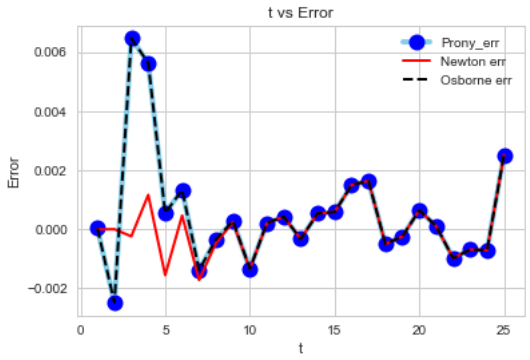


Figure 8: Error

2.8.2 CI of Parameters estimated by Gauss-Newton method

- CI of $\alpha_1 = (0.279191, 4.406988)$
- CI of $\alpha_2 = (8.407516, 12.535313)$
- CI of $\beta_1 = (-3.201222, 0.926574)$
- CI of $\beta_2 = (-6.084447, -1.956650)$

2.8.3 CI of Parameters estimated by Osborne's method

- CI of $\alpha_1 = (0.901507, 5.029304)$
- CI of $\alpha_2 = (3.251312, 7.379109)$
- CI of $\beta_1 = (-3.309815, 0.817981)$
- CI of $\beta_2 = (-6.186193, -2.058396)$

3 Results

The codes to solve the problems in the project is named as Non linear regression.ipynb, etc which are self-explanatory names. In this section, I am going to discuss these codes along with the results that were obtained by running them on my system. First of all, I have imported data using pandas lib function (pd.read_csv) then solved the each of the question step by step as it was asked. In the last, I have created a dataFrame named: final_data which contain
cols = (t, y, Prony, Gauss Newton, Osborne, Prony_err, Newton_err, Osborne_err)

- Prony : Predicted out by prony's method
- Gauss Newton: Predicted out by Gauss Newton's method
- Osborne: Predicted out by Osborne's method
- Prony_err : Error Predicted out by prony's method
- Gauss Newton_err: Error Predicted out by Gauss Newton's method
- Osborne_err: Error Predicted out by Osborne's method

Method	Error
Prony	0.000114%
Gauss Newton	0.000010%
Osborne	0.000184%